Killara HIGH SCHOOL

Student Number

2014

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

** July 2014

General Instructions

- Reading time – 5 minutes
- Working time –3 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 100

Section I - Pages 2 - 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 6 - 12

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Question 1 – 10
Allow about 15 minutes for this section

Use the multiple – choice answer sheet for Questions 1 – 10

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1 If $\dfrac{1}{2+\sqrt{3}} = a + b\sqrt{3}$ then

(A) a = 2 and b =1

(B) a = 2 and b =-1

(C) a = -2 and b =-1

(D) a = 4 and b =1

2 Which of the following is true for the equation $7x^2 - 5x + 2 = 0$

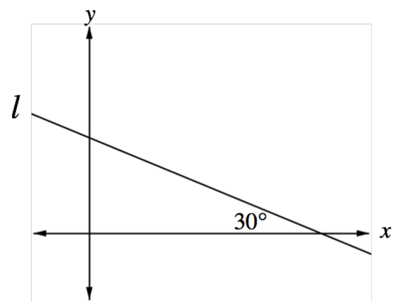
(A) No real roots

(B) One real root

(C) Two real roots

(D) Three real roots

3 The diagram shows the line l.



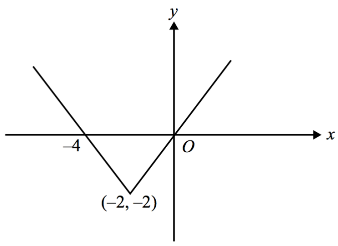
What is the slope of the line l?

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

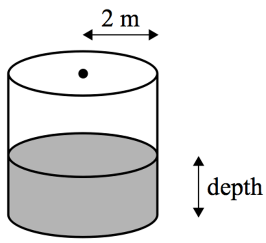
(C) $\dfrac{\sqrt{3}}{3}$

(D) $-\dfrac{\sqrt{3}}{3}$

4	What is the solution to the inequality $\lvert 2x+1\rvert \le 3$ (A) $x \ge 1$ or $x \ge -2$ (B) $x \ge 1$ or $x \ge 2$ (C) $x \le 1$ or $x \ge -2$ (D) $x \le 1$ or $x \ge 2$
5	What is the derivative of $\dfrac{x}{\sin x}$? (A) $\dfrac{\sin x + x\cos x}{\sin^2 x}$ (B) $\dfrac{\sin x - x\cos x}{\sin^2 x}$ (C) $\dfrac{x\sin x - \cos x}{\sin^2 x}$ (D) $\dfrac{x\cos x - \sin x}{\sin^2 x}$
6	 The rule of the function whose graph is given above is: (A) $y = \lvert x-2\rvert + 2$ (B) $y = \lvert x+2\rvert - 2$ (C) $y = \lvert 2-x\rvert - 2$ (D) $y = \lvert 2+x\rvert + 2$

7	A bag contains the numbers 1 – 20. A single number is drawn. What is the probability that it is a multiple of 2 OR a multiple of 5? (A) $\frac{3}{5}$ (B) $\frac{9}{10}$ (C) $\frac{1}{2}$ (D) $\frac{8}{25}$
8	The domain of the function $y = \log_e(2x-1)$ is: (E) all real x , $x \neq \frac{1}{2}$ (F) all real x , $x \geq \frac{1}{2}$ (G) all real x , $x \leq \frac{1}{2}$ (H) all real x , $x > \frac{1}{2}$

9 Water is being poured into a long cylindrical storage tank of radius 2 metres, with its circular base on the ground, at a rate of 2 cubic metres per second.



(A) $\dfrac{1}{8\pi}$

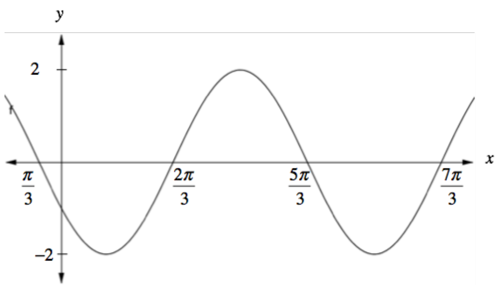
(B) $\dfrac{1}{4\pi}$

(C) $\dfrac{1}{2\pi}$

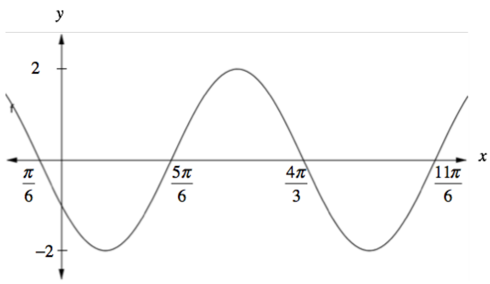
(D) 2π

10 Which of the following is a graph of $y = -2\cos\left(2x - \dfrac{\pi}{3}\right)$

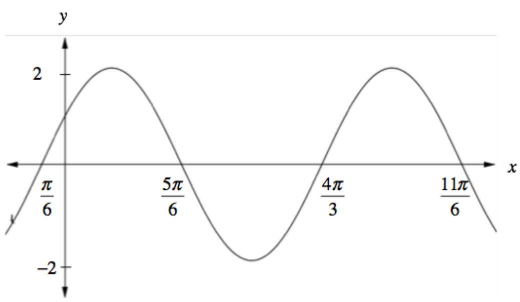
(A)



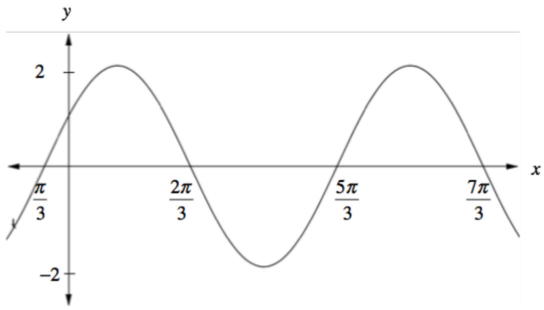
(B)



(C)



(D)



Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a)	Consider the line $x+2y-4=0$ Sketch this line showing clearly the intercepts on both axes.	**2**
(b)	Solve $\frac{5}{2a}-\frac{3}{2a}=\frac{1}{2}$	**2**
(c)	Solve $\lvert x-6\rvert=11$	**2**
(d)	Find the equation of the tangent to the curve $x^2=4y$ at the point (6, 9)	**2**
(e)	Solve for θ where $0\le\theta\le\pi$, $3\tan^2\theta=1$	**2**
(f)	Solve $\log_2 7=x$ giving your answer to 3 significant figures	**1**
(g)	Differentiate (i) $4x\sin\frac{x}{4}$ (ii) $\frac{4xe^x+3x^2}{x}$	 **2** **2**

Question 12(15 marks) Use a SEPARATE writing booklet

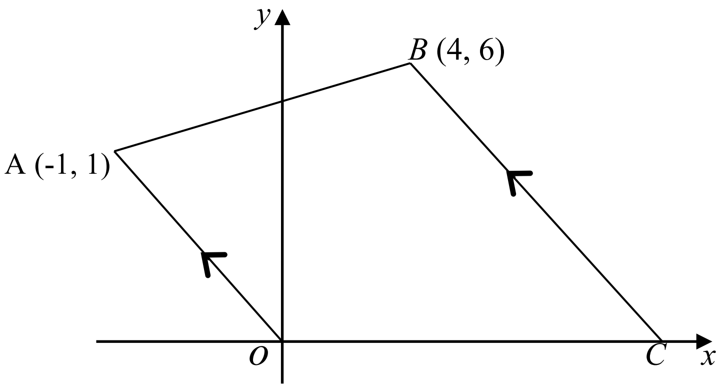
(a)

(iii) Find $\int \pi \, dx$. **1**

(iv) Find $\int \frac{6}{3x+1} dx$. **2**

(v) Find $\int_0^1 (\sqrt{x} - \sqrt[3]{x}) dx$ **2**

(b) In the diagram, $OABC$ is a trapezium with $OA \| CB$. The coordinates of O, A and B. The coordinates of O, A, and B are (0, 0), (-1, 1) and (4, 6) respectively.



NOT TO SCALE

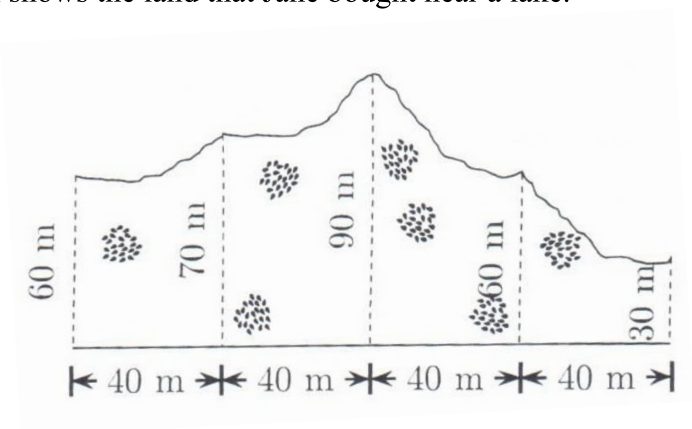
(i) Calculate the distance of OA. **1**

(ii) Find the equation of the line BC, and hence find the coordinates of C. **2**

(iii) Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$. **2**

(c) Shade the region in the plane defined by $y < 3$ and $y \geq x^2 - 4x + 3$. **2**

(d) The diagram shows the land that Jane bought near a lake.

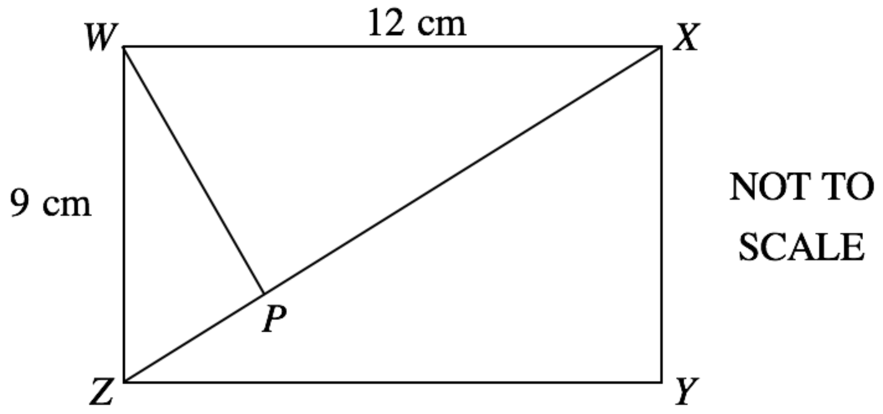


(i) Use the trapezoidal rule with 5 function values to find an estimate for the area of this land **2**

(ii) Is the area obtained in the previous part smaller or larger than the exact area? Give reasons for your answer. **1**

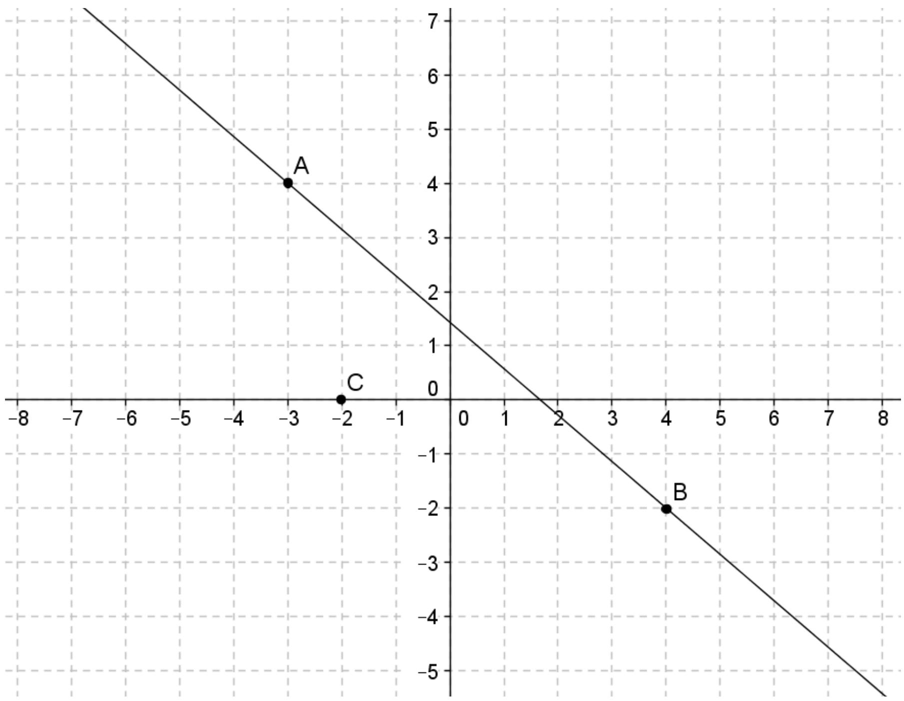
End of Question 12

Question 13(15 marks) Use a SEPARATE writing booklet

(a)	Find the values of a, b c for which $a+b(x+2)+cx(x+2)\equiv 6x^2+x-2$	**3**
(b)	 In the diagram above, WXYZ is a rectangle with WX=12cm and WZ=9cm. WP is perpendicular to XZ. Copy the diagram onto your worksheet (i) Find the length of XZ (ii) Prove that Δ WXP is similar to ΔZXW. (iii) Hence find the length of XP.	 **1** **3** **1**
	Question 13 continues on page 9	

Question 13 (continued)

(c)



On a set of coordinate axes, A is the point (-3,4) and B is the point (4,-2).

i.) Show that the equation of the line AB is $6x+7y-10=0$ **2**

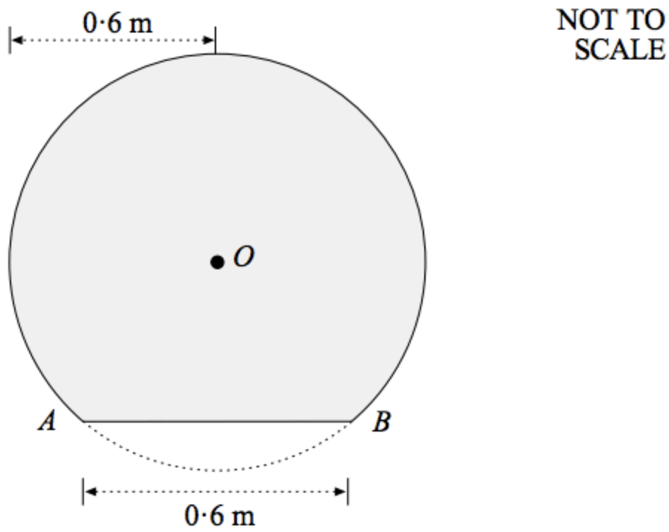
ii) What is the equation of the line parallel to AB through the point C (-2,0) **2**

iii) What is the shortest distance from C to the line AB. Give your answer in exact form. **2**

iv) Write down the equation of the circle which has a centre C and AB as a tangent. **2**

End of Question 13

Question 14(15 marks) Use a SEPARATE writing booklet

(a)	Toby has an old laptop that causes an error when he turns it on 15% of the time. He takes it to a repair technician who asks him to turn the computer on and off to demonstrate the error. (i) What is the probability the computer doesn't demonstrate the error on the first three attempts to start it? (ii) How many times must Toby restart the computer to have a greater than 75% chance of demonstrating the error at least once to the technician?	 **1** **2**
(b)	 A table top is in the shape of a circle with a small segment removed as shown. The circle has a centre *O* and radius 0.6 metres. The length of the straight edge *AB* is also 0.6 metres. (i) Explain why $\angle AOB = \dfrac{\pi}{3}$ (ii) Find the area of the table-top.	 **2** **2**

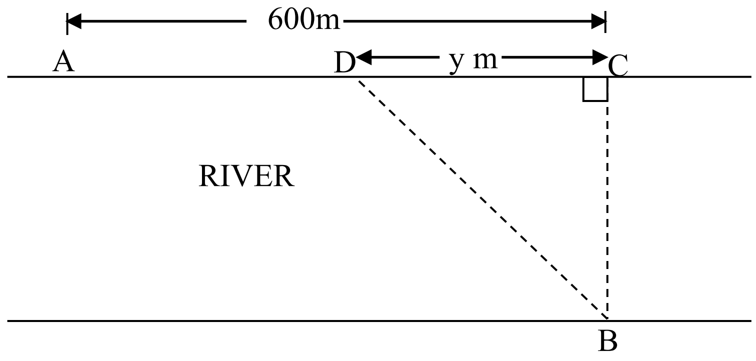
(c)	Find the exact area bounded by $y = \sin(2x)$ and the x-axis between $x = 0$ and $x = \pi$	**3**
(d)	A new species of super ant from the Black Sea (Lasius Neglectus) is spreading throughout the world's ports. It rapidly supplants local ant populations and takes over, hosting 10 to 100 times the number of worker ants as local species. New queens often stay with colony making yet more workers. A small colony was found in a backyard in Sydney, estimated at only 10 queens. After 3 years the nest covers an entire suburb involving an estimated 800 queens. Their growth takes the form $P = P_0 e^{kt}$ where k is a positive constant and P_0 is the original population discovered at time $t = 0$ years. (i) Find the value of k. (ii) The largest colony ever found had 35000 queens. How long until the Sydney colony reaches that many queens? (iii) What could limit the accuracy of this formula for large values of t?	 **2** **2** **1**
	End of Question 14	

Question 15(15 marks) Use a SEPARATE writing booklet

(d)	A point P moves in a straight line, so that its displacement from the origin after t seconds is given by: $x = 3e^{-5t} + 3t - 2$	
	(i) Find the initial displacement, velocity and acceleration of the particle.	**3**
	(ii) Find the time at which the particle is stationary and hence find its displacement and acceleration at that time.	**3**
	(iii) Find an expression for the acceleration of the particle in terms of tis velocity.	**2**
	(iv) Describe the motion of particle P as $t \to \infty$	**1**
(e)	The equation below refers to the filtering cycle of a pump in Helen's garden. The flow rate of the volume of water that the filter pumps water into and out of a pond in litres per minute , is given by $\frac{dV}{dt} = 20\sin\frac{\pi}{35}t$.	
	(i) If the pump started at 8.55pm, what is the first time after 8.55 pm at which the flow rate is zero?	**2**
	(ii) If the pond is initially empty find an expression for the volume, V, of water in the pond after t minutes.	**2**
	(iii) Find the maximum volume of water in the pond during the filtering cycle. Leave your answer in terms of π.	**2**
	End of Question 14	

Question 16 (15 marks) Use a SEPARATE writing booklet

(f)



Jackson's farm house, at 'A', is on the edge of a river 200 metres wide. His horse stables, at 'B', is on the other side of the river. Jackson wants to run an electricity cable from his farm house to the stables.

It costs $1,750 per 100m to lay cables along the river's edge and $2,900 to lay the cable under water.

'C' is the point on the river closest to 'B' and the distance AC is 600

The point 'D' on the rivers bank is at a distance of 'y' metres from 'C'.

(i) Find the total cost in laying the cable in a straight line from A to C and then in a straight line from C to B. **1**

(ii) What is the cost of running the cable directly from A to B. **1**

(iii) Let $F be the total cost of laying the cable in a straight line from A to D, and then in a straight line from D to B. **2**

Show that $F = 1750\ (6 - 0.01y)\ + 290\ (\sqrt{400 + 0.01y^2}\ )$

(iv) Find the minimum cost of laying the cable. **4**

(v) A week before Jackson accepted the minimum costs as determined in part (iii), new technology revealed that the cost of laying the cable underwater was reduced to $2000 per 100 metres. **2**

In this case determine the path for laying the cable in order to minimise the costs.

(g)	The average monthly temperature, T^0 C, in Canberra can be modelled by the formula $T = 7\sin(nx + 1.5) + 13$, where n = a constant value and x = the number of the month of the year (that is, January =1, February =2....) (i) According to the model, what are the maximum and minimum average monthly temperatures in Canberra? (ii) The period of the function is 12. Determine the value of n correct to 2 decimal places. (iii) Which month has the lowest average monthly temperature?	**2** **1** **2**

Yr 12 Mathematics Trial 2014

Answers.

1 B

2 A

3 D

4 C

5 B

6 B

7 A

8 D

9 C

10 B

YR 12 TRIAL HSC 2014 - MATHEMATICS

SOLUTIONS / COMMENTS

QUES 11

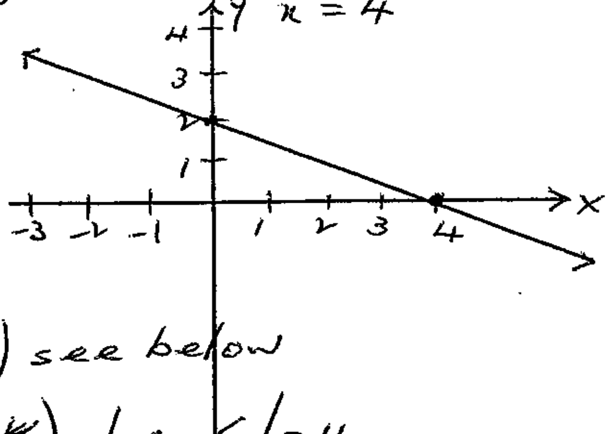
a) $x + 2y - 4 = 0$

$x = 0, \quad 2y - 4 = 0$

$y = 2$

$y = 0, \quad x - 4 = 0$

$x = 4$



① mark for showing intercepts algabraically

① mark for sketch of decent size, clearly showing intercepts.

* Question well done.

b) see below

c) $|x - 6| = 11$

$x - 6 = 11$ or $x - 6 = -11$

$x = 17$ or $x = -5$

① mark for each solution = 2 marks

* Question generally well done

d) $x^2 = 4y \quad (6, 9)$

$y = \frac{x^2}{4}$

$y' = \frac{2x}{4} = \frac{x}{2}$

$f'(6) = \frac{6}{2} = 3$

equation, $y - y_1 = m(x - x_1)$

$y - 9 = 3(x - 6)$

$y - 9 = 3x - 18$

$0 = 3x - y - 9$

① mark for correct derivative / value for gradient

① mark for equation of tangent

* some students still not taking derivative and substituting correctly.

b) $\frac{5}{2a} - \frac{3}{2a} = \frac{1}{2}$

$\frac{2}{2a} = \frac{1}{2}$

$\therefore a = 2$

① mark for simplification

① mark for solution

* well done.

e) $3\tan^2\theta = 1 \quad 0 \le \theta \le \pi$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

for $0 \le \theta \le \pi$, θ is in 1st, 2nd quadrants only.

$\therefore$ basic/acute $\theta = \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \frac{\pi}{6}, \frac{5\pi}{6}$

(1) mark for $\pm\frac{1}{\sqrt{3}}$

(1) mark for correct solutions.

* many students lost negative sign i.e. only gave $\tan\theta = \frac{1}{\sqrt{3}}$
* some students did not use radians
* some students used $0 < \theta \le 2\pi$

f) $\log_2 7 = x$

$x = \frac{\log 7}{\log 2}$

$= 2.807354922$

$= 2.81$ (3 s.f.)

(1) mark for correct value, with no penalty for significant figures.

* students need to check/know difference between s.f. and decimal places.

g) i) $\frac{d}{dx} 4x\sin\frac{x}{4}$

(PRODUCT RULE)

$= VU' + UV'$

$= \sin\frac{x}{4} \cdot 4 + 4x \cdot \frac{1}{4}\cos\frac{x}{4}$

$= 4\sin\frac{x}{4} + x\cos\frac{x}{4}$

(1) mark for each part i.e. VU', UV' = 2 marks.

* several students cancelled incorrectly i.e. $\left(\sin\frac{x}{4}\right) \times 4 \ne \sin x$!
* also $\frac{d}{dx}\sin\frac{x}{4} = \frac{1}{4}\cos\frac{x}{4}$ not $4\cos\frac{x}{4}$

ii) $\frac{d}{dx}\frac{4xe^x + 3x^2}{x}$

$= \frac{d}{dx} 4e^x + 3x$

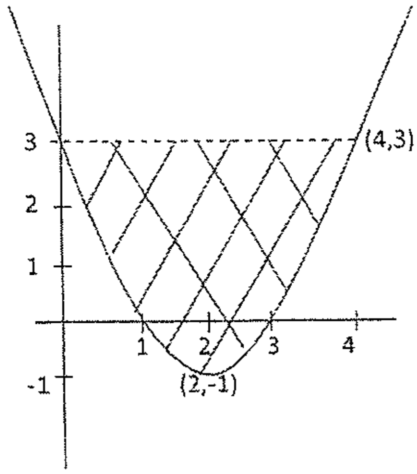
$= 4e^x + 3$ (1) (1)

(1) mark each part, although

* most students made this question complicated by using the quotient rule instead of simplifying first!

Question 12	Answer	Mark
a. i)	$\int \pi\, dx = \pi x + C$	(1) mark to integrate the equation
ii)	$\int \frac{6}{3x+1}dx = 2\int \frac{3}{3x+1}dx$ $= 2\ln(3x+1) + C$	(1) mark for working. (1) mark for correct answer.
iii)	$\int_0^1 (\sqrt{x} - \sqrt[3]{x})dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{4}{3}}\right]_0^1$ $=\left(\frac{2}{3}(1)^{\frac{3}{2}} - \frac{3}{4}(1)^{\frac{4}{3}}\right) - \left(\frac{2}{3}(0)^{\frac{3}{2}} - \frac{3}{4}(0)^{\frac{4}{3}}\right)$ $=\left(\frac{2}{3} - \frac{3}{4}\right) - (0-0)$ $=-\frac{1}{12}$	(1) mark for working. (1) mark for correct answer.
b. i)	$OA = \sqrt{(0+1)^2 + (0-1)^2}$ $= \sqrt{2}$ units	(1) mark for working with correct answer
ii)	$BC \parallel AO$ $\therefore m_{(BC)} = m_{(AO)} = -1$ Equation of the line BC: $y - y_1 = m(x - x_1)$, given $B\,(4, 6)$ $y - 6 = -1(x-4)$ $y - 6 = -x + 4$ $x + y - 10 = 0.$ When $y = 0$, $x = 10$. $\therefore C(10, 0)$.	(1) mark for finding equation of the line (1) mark for finding point $C(10,0)$.
iii)	$d = \frac{\lvert ax_1 + by_1 + c\rvert}{\sqrt{a^2+b^2}}$. The equation of BC is $x + y - 10 = 0$ and (x_1, y_1) is $O\,(0, 0)$. $\therefore d = \frac{\lvert 1\times 0 + 1 \times 0 - 10\rvert}{\sqrt{1^2+1^2}}$ $= \frac{10}{\sqrt{2}}$ $= \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= 5\sqrt{2}$ units	(1) mark for applying perpendicular distance formula (1) mark for correct answer.

c.



1 mark for graph

1 mark for region.

d. i)

We have the table of values

x	0	40	80	120	160
y	60	70	90	60	30
	y_0	y_1	y_2	y_3	y_4

Trapezoidal rule:

$$A \approx \frac{h}{2}(y_0 + 2(y_1 + y_2 + y_3) + y_4)$$

$$A \approx \frac{40}{2}(60 + 2(70 + 90 + 60) + 30)$$

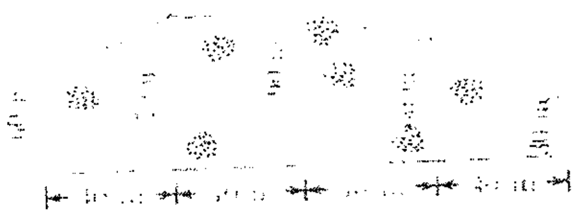
$$= 10\,600 \text{ m}^2.$$

1 mark for applying Trapezoidal rule

1 mark for correct answer

ii)

By joining the tips of the boundaries of the sections of the land, we form the trapeziums that the trapezoidal rule takes the sum of their areas.

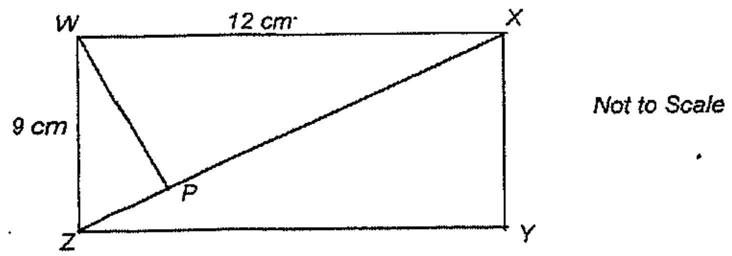


It can be seen that the Trapezoidal Rule gives an area greater than the actual area (as the sum of the areas of the four trapeziums is greater than the actual area).

1 mark for correct explanation

About 90 % of students attained marks over 80%.

Question 13(15 marks) Use a SEPARATE writing booklet

		Markers Comments
(a)	Find the values of a, b c for which $a+b(x+2)+cx(x+2)\equiv 6x^2+x-2$ $LHS=a+bx+2b+cx^2+2cx$ $=cx^2+x(b+2c)+a+2b$ 1 mark simplifying $LHS=RHS$ $\therefore c=6$ $b+2c=1$ }1 mark equating coefficients $b+12=1$ $\therefore b=-11$ $a+2b=-2$ $a-22=-2$ 1 mark correct value a, b, c $a=20$ Alternative equate coefficients to find c=6 Pur x=-2 then $a=6(-2)^2+(-2)-2$ $a=20$ put x=-1 both sides solve for b $20+b(3)+6(3)=6+1-2$	**Mostly done well if students remembered to equate coefficients some errors simplifying**
(b)	 In the diagram above, WXYZ is a rectangle with WX=12cm and WZ=9cm. WP is perpendicular to XZ. Copy the diagram onto your worksheet (i) Find the length of XZ (ii) Prove that Δ WXP is similar to ΔZXW. (iii) Hence find the length of XP. (i)	

In XYZ

$XZ = \sqrt{(XY^2 + YZ^2)}$

$= \sqrt{(9^2 + 12^2)}$

$= 15cm$ 1 mark correct answer

Done well

In WXP and ZXW

$\angle WPX = 90^o (given)$

$\angle ZWX = 90^o (WXYP \text{ is recatngle given})$

$\angle WXP = \angle ZXW (common\ angle)$

$\therefore\ WXP \,|||\, ZXW\,(two\ corresponding\ angles\ equal)$

}1 mark 2
}90^o angle
with reason

1 mark conclusion
with reason

(ii)

An easy question if you redraw the triangles separately

Must state that <ZWX =90 because a rectangle
Must use 3 letters to name angles, <A is not acceptable.
Using AA or AAA is not accepted reasoning for Similarity
Diagonals in rectangles do not form 45 degree angles

(iv) In WXP and ZWX

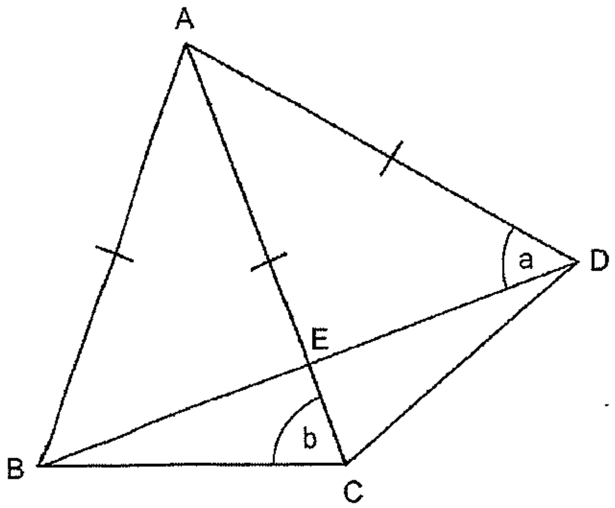
$\frac{XP}{WX} = \frac{XW}{XZ}$(*corresponding sides in similar* s in same ratio)

$XP = \frac{12}{15} \times 12$ 1 mark correct length substituted with reason

$\therefore XP = \frac{144}{15}$

A reason must be given for 1 mark

ABC and *ACD* are congruent isosceles triangles.
Prove that $2b = a + 90^o$



Some very long inefficient solutions. Reasoning was done poorly. Marks were not given for working without reasoning

- **no 90^o angle is given in the question it must be proved.**
- **ADCB is not given as a kite it must be proved**
- **Base angles isosceles triangle is not accepted use "equal angles are opposite equal sides"**

	In ABC and ACD $\angle ABC = \angle ADC = b$ (equal side opposite e qual angles) — *1 mark correct reasoning* $\angle ABD = \angle ABE = \angle ADE = a$ (equal sides oppos ite angles) — *1 mark determining another angle* $\therefore \angle EBC = \angle EDC = b - a$ In BDC $\angle EBC + \angle EDC + \angle DCB = 180^o$ (angle sum of a) $\therefore (b-a)+(b-a)+2b = 180$ $4b - 2a = 180$ — *1 mark final angle sum* $2b = a + 90$	- **Corresponding angles in congruent triangles are the matching angles, not just the equal ones** - **Take care to name angles correctly**

(c)	The graph of the parabola $(x-3)^2 = 12(y-2)$ is shown below. a) Find the coordinates of the vertex Vertex is (3,2) — 1 mark b) Find the coordinates of the focus and the equation of the directrix $(x-3)^2 = 4(3)(y-2)$ $\therefore a = 3$ focus at (3,5) — 1 mark each focus and directrix directrix at $y = -1$ c) Hence or otherwise show that the equation of the circle with centre at the focus passing through the vertex is: $x^2 - 6x + y^2 - 10y + 25 = 0$ $(x - 6x + 9) + (y^2 - 10y + 25) = 9$ — 1 mark expanding and completing square $(x-3)^2 + (y-5)^2 = 9$ this is a circle centre at (3,5) radius=3 — -1 mark for subsequent error passes through (3,2) the vertex	**Done well no working required can be solved by inspection** **Mostly done well** **Mostly done well**

Question 14 — 15 Marks

a) i) P(3 good starts) $= 0.85 \times 0.85 \times 0.85$
$= 0.614125$ ① answer. /1
$\frac{4913}{8000}$

ii) $1 - 0.85^n = 0.75$ — ① for first statement.

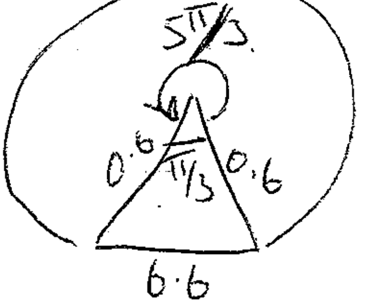
$0.85^n = 0.25$
$n \ln(0.85) = \ln(0.25)$
$n = \frac{\ln(0.25)}{\ln(0.85)}$
$= 8.53004856$ ① for conclusion of 9.

∴ Must start computer 9 times to have a greater than 75% chance of demonstrating the error. /2

b) i) $OA = OB = 0.6$ (radii of same circle)

∴ ΔAOB equilateral as all sides $= 0.6$. ① for reasoning Δ is equilateral
∴ all angles equal & $\pi/3$
∴ $\angle AOB = \pi/3$ /1

ii)



Area of table top = Area of sector + area of Δ

$= \pi r^2 \times \frac{\theta}{2\pi} + \frac{1}{2} \times ab \sin C$. ① for "statement of intent"

$= \pi \times (0.6)^2 \times \frac{5\pi/3}{2\pi} + \frac{1}{2}(0.6)^2 \sin(\pi/3)$
$= 1.098362369$ ① for substitution
~~$= 1.25424694$~~
~~$= 1.3 m^2$ (1dp)~~ $1.1 m^2$ ① for answer /3

Sector ~~$\frac{1}{2}r^2\theta$~~
Segment $\frac{1}{2} r^2 (\theta - \sin\theta)$

arc length $r\theta$

c) $2\int_0^{\pi/2} \sin 2x \, dx$ — ① for breaking up integral to account for Neg. area.

$= 2\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2}$

$= 2 \times -\frac{1}{2}\left[\cos 2x\right]_0^{\pi/2}$ — ① correct integration

$= -\left[\cos \pi - \cos 0\right]$

$= -\left[-1 - 1\right]$ — ① for final solution.

$= -1 \times -2$

$= \underline{2 \text{ units}^2.}$

/3

d) i) $800 = 10e^{3k}$ — ① for substitution

$e^{3k} = 80$

$3k = \ln(80)$

$k = \frac{\ln(80)}{3.} = 1.460675545$ — ① for answer.

/2

ii) $35000 = 10e^{kt}$ — ① for substitution

$3500 = e^{kt}$

$kt = \ln(3500)$

$t = \frac{\ln(3500)}{k}$

$= 5.586811031$ yrs

$\doteqdot 5.6$ yrs (1dp) — ① for answer.

/2

iii) Limited availability of food resources

① for any valid reason.

/1

Q14 $x = 3e^{-5t} + 3t - 2$

i) Initially: $t = 0$

Displacement $x = 3e^{0} + 3\times 0 - 2$
$= 1$ m

Velocity $\dot{x} = -15e^{-5t} + 3$ ①

$t = 0$ $\dot{x} = -15e^{0} + 3$
$= -12$ m/s

Acceleration $\ddot{x} = 75e^{-5t}$

$t = 0$ $\ddot{x} = 75 \times e^{0}$
$= 75$ m/s²

3 marks correctly finds $x, \dot{x}, \ddot{x}$

2 marks correctly finds 2 of the $x, \dot{x}, \ddot{x}$.

1 mark evaluates 1 of the $x, \dot{x}, \ddot{x}$ (consider ecf)

ii) Particle is stationary when $v = 0$

$-15e^{-5t} + 3 = 0$

$e^{-5t} = \frac{1}{5}$, $-5t = \ln\frac{1}{5}$

$t = -\frac{1}{5}(\ln 1 - \ln 5) = \frac{1}{5}\ln 5 = 0.322$ s

$x = 3e^{-5\cdot\frac{1}{5}\ln 5} + 3\left(\frac{1}{5}\ln 5\right) - 2$
$= -0.434$ m

$\ddot{x} = 75e^{-5\times\frac{1}{5}\ln 5}$
$= 15$ m/sec²

3 marks correctly finds the $x, \ddot{x}$ when $v = 0$

2 marks correctly finds the time and x or $\ddot{x}$ OR

1 mark. Finds t when $v = 0$ or evaluates x or $\ddot{x}$ (ecf).

iii) as $t \to \infty$, $e^{-5t} \to 0$

$\therefore$ $\ddot{x} \to 0$, $\dot{x} = v \to 3$ m/s

$x \to$ large positive value, that is it will continue moving in positive/forward direction.

2 marks. correct explanation and shows $t \to \infty$
$\therefore e^{-5t} = \frac{1}{e^{5t}} \to 0$

1 mark any progress towards correct answer

14 b) $\frac{dV}{dt} = 20\sin\frac{\pi}{35}t$

i) $\frac{dV}{dt} = 0$, $20\sin\frac{\pi}{35}t = 0$

$\sin\frac{\pi}{35}t = 0, \pi$

$\frac{\pi}{35}t = \pi$

$t = 35$ minute

$\therefore$ First time after 8:55, $\frac{dV}{dt} = 0$ when

$t = 8{:}55 + 35$

$= 9{:}30$ pm

2 marks correctly finds $t = 35$ min.

1 mark any progress towards solution

ii) $\frac{dV}{dt} = 20\sin\left(\frac{\pi}{35}t\right)$

$V = \int 20\sin\left(\frac{\pi}{35}t\right)dt$

$V = 20\,\frac{-\cos\left(\pi/35\, t\right)}{\frac{\pi}{35}} + C$

$V = \frac{-700}{\pi}\cos\left(\frac{\pi}{35}t\right) + C$

$t = 0$, $V = 0$ $\therefore$ $0 = -\frac{700}{\pi}\cos\left(\frac{\pi}{35}(0)\right) + C$

$C = \frac{700}{\pi} \times 1$ $\therefore V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}t$

2 marks Correctly finds the expression for 'Volume'.

1 mark Correct 'integration' to find 'volume'

iii) Filtering cycle is 35 min. (Part 'i')

$\therefore V = \frac{700}{\pi} - \frac{700}{\pi}\cos\frac{\pi}{35}\cdot 35$

$= \frac{700}{\pi} - \frac{700}{\pi}\cos\pi$

$= \frac{700}{\pi} + \frac{700}{\pi}$

$= \frac{1400}{\pi}$ L

3 marks Correctly evaluate the maximum Volume

2 marks correctly substitute $t = 35$ into formula and some progress towards solution

1 mark Any progress towards solution

Question 16

(a)

(i) $\text{Cost} = (12 \times 600) + (29 \times 450)$
$= \$20,250$ 1

1 for answer. (with working)

(ii) $AB^2 = 600^2 + 450^2$
$AB = 750\text{m}$
$\text{Cost} = 29 \times 750$
$= \$21,750$ 1

1 for answer (with working)

(iii) $DB = \sqrt{y^2 + 450^2}$ $AD = 600 - y$
$\text{Cost} = ~~29~~ \; 12 \times (600 - y) + 29 \times \sqrt{y^2 + 450^2}$
ie $F = 12(600 - y) + 29\sqrt{y^2 + 450^2}$

1 for DB =
1 for AD =

(iv) $F = 12(600 - y) + 29(y^2 + 450^2)^{\frac{1}{2}}$
$\frac{dF}{dy} = -12 + \frac{29}{2}(y^2 + 450^2)^{-\frac{1}{2}} \times 2y$
$= -12 + 29y(y^2 + 450^2)^{-\frac{1}{2}}$ 1

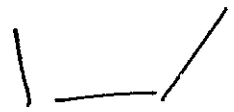
$\frac{dF}{dy} = 0$ when $\frac{29y}{(y^2 + 450^2)^{\frac{1}{2}}} = 12$
$\frac{841y^2}{y^2 + 450^2} = 144$
$841y^2 = 144y^2 + 29,160,000$
$697y^2 = 29,160,000$
$y^2 = ~~42199.71056~~ \; 41836.44$
$y = 204.5\text{m}$ 1

1 for working towards $\frac{dF}{dy} = -12 + 29y(y^2+ \ldots$

1 for working towards $y = 204.5\text{m}$

Testing $y = 205.4$

y	200	204.5	210
$\frac{dF}{dy}$	-0.22	0	0.26



1

1 for showing $y = 204.5$ is a Min Value

$\therefore$ Minimum Cost occurs when

$y = 204.5$

$$\text{Cost} = F = 12 \times (600 - 205) + 29 \times \sqrt{205^2 + 450^2}$$

$$= \$19{,}080.34$$ 1

1 for cost

v) F becomes

$$F = 12(600 - y) + 15(y^2 + 450^2)^{\frac{1}{2}} \quad \text{——} (1)$$

$$\therefore \frac{dF}{dy} = -12 + \frac{15}{2}(y^2 + 450^2)^{-\frac{1}{2}} \times 2y$$

$$= -12 + 15y(y^2 + 450^2)^{-\frac{1}{2}}$$

$$\frac{dF}{dy} = 0 \text{ when } \frac{15y}{(y^2+450^2)^{\frac{1}{2}}} = 12$$

$$225y^2 = 144(y^2 + 450^2)$$

$$81y^2 = 29\,160\,000$$

$$y^2 = 360{,}000$$

ie $y = 600\text{m}$ 1

It is not enough to show A to B is the cheapest of 3 scenarios. <u>Must</u> show working that leads to AB being cheapest out of <u>all</u> possibilities

$\therefore$ He will run the cable directly from A to B ($y = 600$ means $AC = 0$)(or $AD = 0$)

1 $y = 600$

$$\text{Minimum Cost} = 15 \times (600^2 + 450^2)^{\frac{1}{2}}$$

(from (1))

$$= \$11{,}250$$ 1

1 for saying and showing AB is the Min path

(b)

(i) Max Value of $\sin(0.52x+1.5)=1$

$\therefore$ Max $T = 7\times1+13$

$= 20°C$ 1

(ii) Min Value of $\sin(0.52x+1.5)=-1$

$\therefore$ Min $T = 7\times -1+13$

$= 6°C$ 1

(ii) $T = 7\sin(0.52x+1.5)+13$

Min Temp is 6°

$\therefore 6 = 7\sin(0.52x+1.5)+13$ 1

$\frac{-7}{7} = \sin(0.52x+1.5)$

$0.52x = \sin^{-1}(-1) - 1.5$

~~$\sin^{-1}(-1) =$~~

$x = \left(\frac{3\pi}{2} - 1.5\right) \div 0.52$ 1

$x \doteq 6$

$\therefore$ June has the lowest average temp

(iii) Find when the hottest month in Canberra is ie when $T = 20$ and note that this is ~~the~~ also the coldest month in New York

$-A\sin(0.52x+1.5)+13$ 1

No marks for no working.

No marks for wrong working but ~~marks~~ answers correct

~~i 20° 6~~

i/ If done via a table 1 out of 2.

ii If read from table 1 out of

In i.

1 for working towards $T=20$ (Max)

1 for working toward $T=6$ (Min)

ii 1 for showing $6=7\sin(0.52x+1.5)+$

1 for ~~show~~ solving to $x=6$ and stating June has lowest av temp

iii 1 for showing stating $-A\sin(\ )$

i (b)

$T = 7\sin(0.52x + 1.5) + 13$

$T' = 7 \times 0.52\cos(0.52x + 1.5)$

$T' = 3.64\cos(0.52x + 1.5)$

$T'' = -3.64 \times 0.52\sin(0.52x + 1.5)$

$= -1.8928\sin(0.52x + 1.5)$

$T' = 0$ when $\cos(0.52x + 1.5) = \frac{\pi}{2}, \frac{3\pi}{2}$

$0.52x = \frac{\pi}{2} - 1.5$, $0.52x = \frac{3\pi}{2} - 1.5$

$x = \frac{\frac{\pi}{2} - 1.5}{0.52} = 0.1361$ $\quad$ $x = \frac{\frac{3\pi}{2} - 1.5}{0.52}$, $x = 6.1777$

when $x = 0.1361$

$T'' = -1.89$

$\therefore$ Max V when $x = 0.1361$

~~when~~

$\therefore T = 7\sin(0.52 \times 0.1361 + 1.5) + 13$

$= 20$

is the Max T

when $x = 6.1777$

$T'' = 1.8928$

$\therefore$ Min Value when $x = 6.1777$

$T = 7\sin(0.52 \times 6.1777 + 1.5) + 13$

$= 6$ is the Min T

ii From part i $x = 6.1777$ (Min Value via T'')

gives $T = 6$ (Min Temp

$\therefore$ June gives the lowest.

Alternative answer for b i ii

1 for both x values and showing they give Max + Min Values via T'' or table.

1 for answers (must be supported)

1 Must have shown that x = 6.1777 does give a Min Value

1 for answer (only if can be supported)